SOLVED EXAMPLES

Ex.1 If $f(x) = \begin{cases} x^2+2, x \geq 1 \\ 2x+1, x < 1 \end{cases}$, then $\lim\limits_{x\to 1} f(x)$ equals -

(A) 1 (B) 2

(C) 3 (D) Does not exist

Sol. $\lim\limits_{x\to 1-0} f(x) \lim\limits_{h\to 0} = [2(1–h)+1]= 3$

$\lim\limits_{x\to 1+0} f(x) = \lim\limits_{h\to 0} [(1+h)^2 + 2] = 3$

$\therefore$ LHL = RHL, so $\lim\limits_{x\to 1} f(x) = 3$. **Ans.[C]**

Ex.2 $\lim\limits_{x\to 0} \frac{1+e^{-1/x}}{1-e^{-1/x}}$ is equal to -

(A) 1 (B) –1

(C) 0 (D) Does not exist

Sol. LHL = $\lim\limits_{h\to 0} \frac{1+e^{1/h}}{1-e^{1/h}}$

$= \lim\limits_{h\to 0} = \frac{e^{-1/h}+1}{e^{-1/h}-1} - 1$

RHL = $\lim\limits_{h\to 0} = \frac{1+e^{-1/h}}{1-e^{-1/h}} = \frac{1+0}{1-0} = 1$

LHL $\neq$ RHL, so given limit does not exist. **Ans.[D]**

Ex.3 $\lim\limits_{x\to\infty} \frac{2x^2+3x}{3x^2+4}$ equals -

(A) 1/2 (B) 2/3

(C) 3/4 (D) 0

Sol. $= \lim\limits_{x\to\infty} \frac{2+(3/x)}{3+(4/x^2)} = \frac{2}{3}$ **Ans.[B]**

Ex.4 $\lim\limits_{x\to\infty} \left(\sqrt{x^2+1} - x\right)$ equals -

(A) –1 (B) 0

(C) 1 (D) None of these

Sol. Limit = $\lim\limits_{x\to\infty} x\left[\left(1+\frac{1}{x^2}\right)^{1/2} - 1\right]$

$= \lim\limits_{x\to\infty} x\left[1+\frac{1}{2x^2} - \frac{1}{8x^4} + ... - 1\right]$

$= \lim\limits_{x\to\infty} \left[\frac{1}{2x} - \frac{1}{8x^3} + ...\right] = 0.$ **Ans.[B]**

Ex.5 $\lim\limits_{x\to -1} \left(\frac{x^2-1}{x^2+3x+2}\right)$ is equal to-

(A) –2 (B) 1/2

(C) 0 (D) 1

Sol. Limit = $\lim\limits_{x\to -1} \frac{(x-1)(x+1)}{(x+2)(x+1)} = \frac{-1-1}{-1+2} = -2$

Ans.[A]

Ex.6 $\lim\limits_{x\to a} \left[\frac{x^2-(a+1)x+a}{x^3-a^3}\right]$ is equal to -

(A) $\frac{a-1}{3a^2}$ (B) a – 1

(C) a (D) 0

Sol. $\lim\limits_{x\to a} \left[\frac{x^2-(a+1)x+a}{x^3-a^3}\right] \left(\frac{0}{0}\text{form}\right)$

$= \lim\limits_{x\to a} = \frac{2x-a-1}{3x^2} = \frac{a-1}{3a^2}$

(D.L.Hospital rule) **Ans.[A]**

Ex.7 $\lim\limits_{x\to 3} \frac{x-3}{|x-3|}$, is equal to -

(A) 1 (B) –1

(C) 0 (D) Does not exist

Sol. LHL = $\lim\limits_{h\to 0} \frac{(3-h)-3}{|(3-h)-3|}$

$= \lim\limits_{h\to 0} \frac{-h}{|-h|} = -1$

RHL = $\lim\limits_{h\to 0} \frac{(3+h)-3}{|(3+h)-3|}$

$= \lim\limits_{h\to 0} \frac{h}{|h|} = 1$

LHL $\neq$ RHL, so limit does not exist.**Ans.[D]**

Ex.8 If $f(x) = \frac{x+|x|}{x}$, then $\lim\limits_{x\to 0} f(x)$ equals-

(A) 2 (B) 0

(C) 1 (D) Does not exist

Sol. LHL = $\lim\limits_{h\to 0} \frac{-h+|h|}{-h} = \lim\limits_{h\to 0} (0) = 0$

RHL = $\lim_{h\to 0} \frac{h+|h|}{h} = 2$

LHL ≠ RHL ⇒ does not exist. **Ans.[D]**

Ex.9 $\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$ is equal to -

(A) 1/2 (B) 2

(C) 1 (D) 0

Sol. Limit = $\lim_{x\to 0} \frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{(1+x)-(1-x)}$

$= \lim_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{2} = 1.$ **Ans.[C]**

Ex.10 $\lim_{x\to 0} \frac{xe^{x}-\log(1+x)}{x^2}$ equals -

(A) 1/2 (B) 1

(C) 3/2 (D) 2

Sol. $\lim_{x\to 0} \frac{x\left(1+x+\frac{x^2}{2!}+....\right)-\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+....\right)}{x^2}$

$= \lim_{x\to 0} \left(\frac{3}{2}+\frac{1}{6}x+...\right) = 3/2$ **Ans.[C]**

Ex.11 The value of $\lim_{x\to 0} \left[\frac{1}{x^2}-\frac{1}{\sin^2 x}\right]$ is -

(A) –1/2 (B) 1/2

(C) –1/3 (D) 1/3

Sol. Limit = $\lim_{x\to 0} \frac{\sin^2 x - x^2}{x^2.\sin^2 x}$

$= \lim_{x\to 0} \frac{\left(x-\frac{x^3}{3!}+...\right)^2 - x^2}{x^2\left(x-\frac{x^3}{3!}+...\right)^2}$

$= \lim_{x\to 0} \frac{x^2-\frac{1}{3}x^4+....-x^2}{x^4\left(1-\frac{x^2}{3!}+...\right)^2} = -1/3$ **Ans.[C]**

Ex.12 $\lim_{x\to 0} \frac{\tan 2x - x}{3x-\sin x}$ equals-

(A) 2/3 (B) 1/3

(C) 1/2 (D) 0

Sol. The given limit is in the form , therefore applying L 'Hospital's rule, we get

Limit = $\lim_{x\to 0} \frac{2\sec^2 2x-1}{3-\cos x} = \frac{2-1}{3-1} = \frac{1}{2}$ **Ans.[C]**

Ex.13 $\lim_{x\to 0} \frac{\sin x+\log(1-x)}{x^2}$ is equal to -

(A) 0 (B) 1/2

(C) –1/2 (D) Does not exist

Sol. It is in 0/0 form, so using Hospital rule, we have

Limit = $\lim_{x\to 0} \frac{\cos x - \frac{1}{1-x}}{2x}$ (0/0 form)

$= \lim_{x\to 0} \frac{-\sin x - \frac{1}{(1-x)^2}}{2} = -1/2$ **Ans.[C]**

Ex.14 $\lim_{x\to\infty} \frac{\sin x}{x}$ equals -

(A) 1 (B) 0

(C) ∞ (D) Does not exist

Sol. $\lim_{x\to\infty} \frac{\sin x}{x}$

$= \lim_{x\to\infty}$ (a finite number between –1 and 1)/∞

= 0 **Ans.[B]**

Ex.15 $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$ is equal to -

(A) e^3 (B) $e^{1/3}$ (C) 1 (D) e

Sol. Limit = $\lim_{x\to 0} \left(\frac{x+x^3/3+....}{x}\right)^{1/x^2}$

$= \lim_{x\to 0} \left(1+\frac{x^3}{3}\right)^{1/x^2}$

[∵ x → 0, so neglecting higher powers of x]

$= \lim_{x\to 0} \left[\left(1+\frac{x^2}{3}\right)^{3/x^2}\right]^{1/3} = e^{1/3}$ **Ans.[B]**

Ex.16 If f(x) = $\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty}$ f(x) equals -

(A) 0 (B) ∞

(C) 1 (D) None of these

Sol. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{\frac{\{1-(\sin x/x)\}}{\{1+(\cos^2 x/x)\}}}$

$= \sqrt{\frac{1-0}{1+0}} = 1.$ **Ans.[C]**

Ex.17 If $G(x) = -\sqrt{25-x^2}$,

then $\lim_{x\to1} \frac{G(x)-G(1)}{x-1}$ equals -

(A) 1/24 (B) 1/5

(C) $-\sqrt{24}$ (D) None of these

Sol. Here $G(1) = -\sqrt{25-x^2} = -\sqrt{24}$

$\therefore$ Given limit

$= \lim_{x\to1} \frac{-\sqrt{25-x^2}+\sqrt{24}}{x-1}$ $\left(\frac{0}{0}\text{form}\right)$

$= \lim_{x\to1} \frac{x}{\sqrt{25-x^2}}$ (By L Hospital ruel)

$= \frac{1}{\sqrt{24}}$ **Ans.[D]**

Ex.18 If f(9) = 9 and f'(9) = 4, then $\lim_{x\to9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to -

(A) 1 (B) 3 (C) 4 (D) 9

Sol. Given limit is in 0/0 form, so using Hospital rule, we get

$\text{Limit} = \lim_{x\to9} \frac{\frac{1}{2\sqrt{f(x)}}.f(x)}{\frac{1}{2\sqrt{x}}}$

$= \frac{f(9).\sqrt{9}}{\sqrt{f(9)}} = \frac{4.3}{3} = 4$ **Ans.[C]**

Ex.19 $\lim_{x\to\infty} \left(\frac{x+2}{x+1}\right)^{x+3}$ is equal to -

(A) 1 (B) e (C) e^2 (D) e^3

Sol. $\text{Limit} = \lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x}.\left(\frac{x+2}{x+1}\right)^{3}.$

$= \lim_{x\to\infty}\left(\frac{1+2/x}{1+1/x}\right)^{x}.\left(\frac{1+2/x}{1+1/x}\right)^{3}$

$= \frac{\lim_{x\to\infty}[(1+2/x)^{x/2}]^2}{\lim_{x\to\infty}(1+1/x)^x} . \lim_{x\to\infty}\left(\frac{1+2/x}{1+1/x}\right)^3$

$= \frac{e^2}{e}.1 = e$ **Ans.[B]**

Ex.20 The value of $\lim_{x\to\infty} \frac{x(\log x)^3}{1+x+x^2}$ is -

(A) 0 (B) 1 (C) –1 (4) 1/2

Sol. $\lim_{x\to\infty} \frac{x(\log x)^3}{1+x+x^2}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{x\to\infty} \frac{(\log x)^3+3(\log x)^2}{1+2x}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{x\to\infty} \frac{3(\log x)^2.\frac{1}{x}+6(\log x).\frac{1}{x}}{2}$

$=. \lim_{x\to\infty} \frac{3(\log x)^2+6\log x}{2x}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{x\to\infty} \frac{6(\log x)\frac{1}{x}+\frac{6}{x}}{2}$

$= 3\lim_{x\to\infty} \frac{\log x+1}{x}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= 3\lim_{x\to\infty} \frac{(1/x)}{1} = 0.$ **Ans.[A]**

Ex.21 $\lim_{x\to0} \frac{\sin x^o}{x}$ is equal to -

(A) 1 (B) π (C) x (D) $\pi/180$

Sol. $\text{Limit} = \lim_{x\to0} \frac{\sin(\pi/180)x}{x}$

$= \lim_{x\to0} \frac{(\pi/180)\cos(\pi/180)x}{1}$

$= \frac{\pi}{180}$ **Ans.[D]**

Ex.22 If $f(x) = \begin{cases} x-1, x<0 \\ 1/4, x=0 \\ x^2, x>0 \end{cases}$ then $\lim_{x\to0} f(x)$ equals -

(A) 0 (B) 1

(C) –1 (D) Does not exist

Sol. Here $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$

and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x-1) = -1$

$\because \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist. **Ans.[D]**

Ex.23 $\lim_{x\to 0} \frac{2^x - 1}{\sqrt{(1+x)} - 1}$ equals -

(A) log 2 (B) 2 log 2

(C) 1/2 log 2 (D) 2

Sol. Given Limit

$= \lim_{x\to 0} \frac{2^x - 1}{\sqrt{(1+x)} - 1} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$

$= \lim_{x\to 0} \frac{2^x - 1}{x} \lim_{x\to 0} (\sqrt{1+x}+1)$

$= 2. \lim_{x\to 0} \frac{2^x \log 2}{1} = 2. \log 2$ **Ans.[B]**

Ex.24 If a,b,c,d are positive real numbers, then $\lim_{n\to\infty}\left(1+\frac{1}{a+bn}\right)^{c+dn}$ is equal to -

(A) $e^{d/b}$ (B) $e^{c/a}$

(C) $e^{(c+d)/(a+b)}$ (D) e

Sol. $\lim_{x\to\infty}\left(1+\frac{1}{a+bn}\right)^{c+dn}$ (1^∞ form)

$= e^{\lim_{x\to\infty}\left(1+\frac{1}{a+bn}-1\right) \times (c+dn)}$

$= e^{\lim_{x\to\infty}\frac{c+dn}{a+bn}}$

$= e^{\lim_{x\to\infty}\frac{\frac{c}{n}+d}{a/n+b}} = e^{d/b}$ **Ans.[A]**

Ex.25 $\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$ equals -

(A) 0 (B) 1 (C) ∞ (D) –1

Sol. Let $y = \lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$

$= \lim_{x\to\infty} (\cot^{-1}x)^{1/x}$

$\therefore \log y = \lim_{x\to\infty} \frac{\log \cot^{-1} x}{x}$ $\left(\frac{\infty}{\infty}\text{form}\right)$

$= \lim_{x\to\infty} -\frac{1}{(1+x^2)\cos^{-1} x}$ ($0x\ \infty$ form)

$= -\lim_{x\to\infty} \frac{(1+x^2)^{-1}}{\cot^{-1} x}$ $\left(\frac{0}{0}\text{form}\right)$

$= -\lim_{x\to\infty} \frac{\frac{-2x}{(1+x^2)^2}}{\frac{-1}{1+x^2}}$ $= -2\lim_{x\to\infty}\frac{x}{1+x^2}$

$= -2\lim_{x\to\infty}\frac{1}{2x} = 0$ $\therefore y = e^0 = 1$.**Ans.[B]**

Ex.26 $\lim_{x\to 0}\frac{x(2^x-1)}{1-\cos x}$ equals -

(1) 0 (2) log 2

(3) 2 log 2 (4) None of these

Sol. The given limit $= \lim_{x\to 0}\frac{2^x-1}{x}.\frac{x^2}{1-\cos x}$

$= \lim_{x\to 0}\frac{2^x-1}{x}\lim_{x\to 0}\frac{x^2}{2\sin^2\frac{x}{2}}$

$= \log x \ .\ 2\lim_{x\to 0}\left(\frac{x/2}{\sin(x/2)}\right)^2$

= 2 log 2. **Ans.[C]**

Ex.27 The value of $\lim_{x\to 0}\left[\frac{a}{x}-\cot\frac{x}{a}\right]$ is -

(A) 0 (B) 1 (C) a (D) a/3

Sol. Given Limit $= \lim_{x\to 0}\left[\frac{a}{x}-\frac{\cos(x/a)}{\sin(x/a)}\right]$

$= \lim_{x\to 0}\left[\frac{a\sin(x/a)-x\cos(x/a)}{x\sin(x/a)}\right]$

$= a\lim_{x\to 0}\left[\frac{a\sin(x/a)-x\cos(x/a)}{x^2}\right] \times \frac{(x/a)}{\sin(x/a)}$

$= a\lim_{x\to 0}\left[\frac{a\sin(x/a)-x\cos(x/a)}{x^2}\right]$ $\left(\frac{0}{0}\text{form}\right)$

$= a\lim_{x\to 0}\left[\frac{\cos(x/a)-\cos(x/a)+(x/a)\sin(x/a)}{2x}\right]$

= 0 **Ans.[A]**